

ANYONS AS SPIN PARTICLES: FROM CLASSICAL MECHANICS TO FIELD THEORY¹

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Abstract

(2+1)-dimensional relativistic fractional spin particles are considered within the framework of the group-theoretical approach to anyons starting from the level of classical mechanics and concluding by the construction of the minimal set of linear differential field equations.

¹Talk given at the Conference “Theories of Fundamental Interactions”, Maynooth, Ireland, May 1995; to be published in “Topics in Quantum Field Theory” (World Scientific, Ed. T. Thcrakian, 1995)

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1 Introduction

The case of (2+1)-dimensional space-time is special from the point of view of spin and statistics: unlike the higher dimensional cases, here spin of particles can take arbitrary values on the real line and statistics interpolating Fermi and Bose statistics is possible. The first peculiarity is coded in the topology of the (2+1)-dimensional Lorentz group and its rotational subgroup manifolds, $\pi_1(SO(2, 1)) = \pi_1(SO(2)) = Z$ [1], whereas the possibility of exotic statistics is realizable due to a nontrivial nature of the fundamental group of the configuration space of N identical particles on the plane: $\pi_1(C_N) = B_N$ [2, 3]. Here Z is an additive group of integer numbers and B_N is a braid group.

The dynamical mechanism, realizing these possibilities at the quantum mechanical level, was proposed by Wilczek [4, 3]. It consists in supplying the system of the charged particles with (singular) magnetic fluxes being concentrated at the positions of the particles. This mechanism is based on the Aharonov-Bohm effect, and, so, it is rooted in the profound principle of non-locality in quantum theory.

Nonlocal nature of fractional (arbitrary) spin particles obeying exotic statistics (anyons) also reveals itself at the level of the quantum field theory, where anyons can be realized with the help of the universal construction, which consists in minimal coupling the Chern-Simons U(1) gauge field to the conserved matter current [3, 5]. The initial formulation of the theory with Chern-Simons gauge field has a local character, but the operators advanced to represent anyons turn out to be realizable as nonlocal operators given on the one-dimensional path – nonobservable half-infinite ‘string’ going to the space infinity [6]. The spin-statistics relation takes place for these anyonic objects [6, 7], but it is not clear how the theory generally is reduced to the local theory of the usual integer and half-integer spin fields in the case when spin of the nonlocal anyonic fields takes integer and half-integer values.

One can try to describe anyons in an alternative way, as we describe spin $s = n/2$ fields, $n = 0, 1, 2, \dots$, not using Chern-Simons U(1) gauge field constructions. But a priori it is clear that such a group-theoretical description of anyons cannot be realized starting from the pseudoclassical mechanics [8]. This ‘no-go theorem’ is rooted in the fact that the quantization of Grassmann variables, which are used there for taking into account spin degrees of freedom, leads to finite-dimensional representations of the (2+1)-dimensional Lorentz group, and, as a result, only integer and half-integer spin fields can be described in this way [9]. Nevertheless, the non-Grassmannian approach can be formulated on the basis of pseudoclassical mechanics [10]. This approach uses usual commuting translation invariant variables instead of anticommuting Grassmann variables and allows us to describe massless [11] and massive [12] particles of arbitrary (fixed) integer and half-integer spin and to realize spin systems with global supersymmetry [13], and, moreover, it naturally leads to the group-theoretical approach for fractional spin particles [14, 15].

The purpose of the present paper is to review some recent results obtained within the framework of the group-theoretical approach to anyons as spin particles.

The paper is organized as follows. Section 2 is devoted to the consideration of the classical mechanics of fractional spin particles. Here we construct the ‘minimal’ classical model of relativistic particle with arbitrary spin, which is quantized in section 3. In section 3 we discuss also the ‘universal’ vector system of linear differential equations and construct the minimal spinor system of linear differential equations. The latter construction is realized

with the help of the deformed (extended) Heisenberg-Weyl algebra. The last section contains concluding remarks.

2 Classical mechanics of fractional spin particles

In 2+1 dimensions, spin $S = p^\mu \mathcal{J}_\mu / \sqrt{-p^2}$ has a pseudoscalar nature and, so, relativistic particle with fixed mass $-p^2 = m^2$ and nonzero spin $S = s \neq 0$ has the same number of degrees of freedom as a massive scalar ($s = 0$) particle. Therefore, one can try to describe spin particle by the minimal set of phase space variables being the coordinates x_μ and canonically conjugate energy-momentum vector p^μ , $\{x_\mu, p_\nu\} = \eta_{\mu\nu}$. Then the total angular momentum vector of the system can be chosen in the form

$$\mathcal{J}_\mu = -\epsilon_{\mu\nu\lambda} x^\nu p^\lambda + j_\mu,$$

generalizing the form of the total angular momentum vector of the scalar particle [16]. Here \mathcal{J}_μ is a vector dual to the total angular momentum tensor, $\mathcal{J}_\mu = -\frac{1}{2}\epsilon_{\mu\nu\lambda}\mathcal{J}^{\nu\lambda}$, the second term $j_\mu = j_\mu(p)$ serves for taking into account a nontrivial spin s , $j_\mu = -sp^\mu/\sqrt{-p^2} + j_\mu^\perp(p)$, $j^\perp p = 0$, and we assume that p^2 is fixed with the help of the mass shell constraint,

$$\phi = p^2 + m^2 \approx 0. \quad (2.1)$$

Generally, the coordinates x_μ can have the Poisson brackets of the form

$$\{x_\mu, x_\nu\} = \epsilon_{\mu\nu\lambda} R^\lambda.$$

The Jacobi identities for the brackets of x_μ and p_ν and classical Poincaré algebra,

$$\{p_\mu, p_\nu\} = 0, \quad \{\mathcal{J}_\mu, \mathcal{J}_\nu\} = -\epsilon_{\mu\nu\lambda} \mathcal{J}^\lambda, \quad \{\mathcal{J}_\mu, p_\nu\} = -\epsilon_{\mu\nu\lambda} p^\lambda,$$

lead to the following most general form of the quantities R^μ and j_μ^\perp , which turn out to be correlated:

$$R_\mu = R_\mu^c + \epsilon_{\mu\nu\lambda} \partial^\nu A^\lambda, \quad R_\mu^c = -sp_\mu/(-p^2)^{3/2}; \quad j_\mu^\perp = -\epsilon_{\mu\nu\lambda} A^\nu p^\lambda,$$

where $A_\mu = A_\mu(p)$ is a set of arbitrary functions having the dimensionality of inverse mass. Only in one special case, when $A_\mu = p_\mu \cdot a(p^2)$ and, therefore, $R_\mu = R_\mu^c$ and $j_\mu^\perp = 0$, the coordinates of the particle $x_\mu = x_\mu^c$ form the Lorentz vector: $\{\mathcal{J}_\mu, x_\nu^c\} = -\epsilon_{\mu\nu\lambda} x^{c\lambda}$, whereas in all other cases x_μ is not a Lorentz covariant object. The Poisson brackets $\{x_\mu^c, x_\nu^c\} = \epsilon_{\mu\nu\lambda} R^{c\lambda}$ mean that at the quantum level there is no representation where the corresponding operators of the coordinates of the particle would be diagonal, and, hence, the *special covariant case is characterized by nonlocalizable coordinates*.

There is another special case characterized by localizable coordinates $x_\mu = x_\mu^l$, $\{x_\mu^l, x_\nu^l\} = 0$. In this case the “gauge field” A_μ is defined by the equation $\partial_\mu A_\nu - \partial_\nu A_\mu = \epsilon_{\mu\nu\lambda} R^{c\lambda}$ meaning that A_μ has a curvature of SO(2,1) monopole. It is given by the expression

$$A_\mu^l = \epsilon_{\mu 0 i} R^{ci} \cdot f(p), \quad f(p) = -\frac{p^0 \cdot p^2}{|p^0| \cdot (|p^0| + \sqrt{-p^2})},$$

which can be considered as a most general solution of the ‘curvature equation’ [16]. One can be convinced that in correspondence with general properties, x_μ^l has complicated transformation properties with respect to the pure Lorentz transformations (boosts), and, hence, is not a Lorentz vector. But due to the property of localizability, these coordinates help to realize covariant operators X_μ^c corresponding to classical coordinates x_μ^c :

$$X_\mu^c = X_\mu^l + A_\mu^l(P).$$

We shall return to the discussion of the quantization of the described formulation of relativistic fractional spin particle, which can naturally be called a *minimal canonical formulation*, in the last section. So, we conclude that within the framework of the minimal formulation the properties of covariance and localizability for the coordinates of arbitrary spin particle cannot be simultaneously incorporated into the theory.

But it can be done via extending the phase space of the system by ‘internal’ phase space variables z_n , $n = 1, \dots, 2N$, $\{z_n, x_\mu\} = \{z_n, p_\mu\} = 0$, and supposing that spin addition depends on these variables: $j_\mu = j_\mu(z_n, p_\nu)$. In this case spin can be fixed by imposing the spin constraint

$$\chi = pj - sm \approx 0. \quad (2.2)$$

Constraints (2.1) and (2.2) form the set of first class constraints of the *extended formulation*, whereas $N - 1$ degrees of freedom, different from the spin one, should be ‘frozen’ by introducing the corresponding number of first or/and second class constraints.

Let us suppose now that j_μ does not depend on p_μ , i.e. $j_\mu = j_\mu(z_n)$, and that coordinates x_μ are localizable, $\{x_\mu, x_\nu\} = 0$. Then it follows that j_μ itself has to satisfy (2+1)-dimensional Lorentz algebra,

$$\{j_\mu, j_\nu\} = -\epsilon_{\mu\nu\lambda} j^\lambda, \quad (2.3)$$

and that x_μ is a Lorentz vector. Therefore, within the framework of the extended formulation we indeed can simultaneously incorporate into the theory the properties of localizability and covariance of the coordinates.

The minimal case of extended formulation is characterized by two internal phase space variables (one degree of freedom frozen by the spin constraint), and it can be realized in the following way. First we note that j^2 lies in the centre of Lorentz algebra (2.3), and, therefore, it can be fixed by putting $j^2 = C = \text{const}$. As a result, we can consider j_μ subject to this condition as the internal variables themselves. Then the topology of the internal phase subspace will be defined by this constant parameter C . In the case $C = -\alpha^2 < 0$, $\alpha^2 \leq s^2$, we have the two sheet hyperboloid, $j_0 = \pm\sqrt{\alpha^2 + j_1^2 + j_2^2}$, as the internal phase subspace, whereas the case $C = \beta^2 \geq 0$ gives a one sheet hyperboloid. The Lagrangian leading to brackets (2.3) and constraints (2.1) and (2.2), has the following form [16]:

$$L_0 = \frac{1}{2e}(\dot{x}_\mu - v j_\mu)^2 - \frac{e}{2}m^2 + smv - \frac{j\xi}{j^2 + (j\xi)^2} \epsilon_{\mu\nu\lambda} \xi^\mu j^\nu \dot{j}^\lambda,$$

where ξ^μ is a constant timelike vector, $\xi^2 = -1$, and e and v are Lagrange multipliers.

On the phase space of the described minimal extended system, one can introduce the vector $\tilde{x}_\mu = x_\mu + (p^2)^{-1} \epsilon_{\mu\nu\lambda} p^\nu j^\lambda$. This vector has, unlike x_μ , zero brackets, $\{\tilde{x}_\mu, \tilde{\chi}\} = 0$, with the spin constraint $\tilde{\chi}$ presented with the help of the mass shell constraint in the dimensionless

form $\tilde{\chi} = jp/\sqrt{-p^2} - s$. So, \tilde{x}_μ is a gauge invariant extension of the initial coordinates x_μ . It has a simple relativistic evolution law, $d\tilde{x}^i/d\tilde{x}^0 = p^i/p^0$. On the other hand, the initial coordinates x_μ reveal more complicated motion as a consequence of a classical analog of the quantum relativistic Zitterbewegung [10], which generally takes place in the system [16]. In this respect the coordinates \tilde{x}_μ are analogous to the Foldy-Wouthuysen coordinates for the Dirac particle. But there is one special case here, which is characterized by the value of the parameter C being correlated with the value of the spin parameter: iff $-j^2 = \alpha^2 = s^2$, there is Lagrange constraint in the system: $\dot{x}^2 - (\dot{x}j)^2/j^2 = 0$. This constraint means that the velocity of the particle is parallel to the vector j_μ and, as a consequence of the spin constraint, to the energy-momentum vector p_μ . So, in this special case x_μ has the same evolution law as \tilde{x}_μ , and, moreover, here the gauge-invariant extension \tilde{x}_μ coincides with x_μ . We shall see below that this case turns out to be special also from the point of view of linear differential equations. In conclusion of classical considerations, we note that the Poisson brackets of the quantities \tilde{x}_μ have the same form (on the surface of constraint (2.2)) as the Poisson brackets of the covariant coordinates x_μ^c in the minimal formulation, whereas the general case of the minimal formulation can be obtained from the extended formulation via reduction of the extended system to the surface of the spin constraint [16].

3 Quantum theory

In correspondence with classical relations (2.3), the operators J_μ must satisfy the algebra of (2+1)-dimensional Lorentz group $\text{SO}(2,1)$ (or $\text{SL}(2,\mathbb{R})$ group locally isomorphic to it):

$$[J_\mu, J_\nu] = -i\epsilon_{\mu\nu\lambda}J^\lambda, \quad (3.1)$$

whereas the first class constraints turn into equations

$$(P^2 + m^2)\Psi = 0, \quad (PJ - sm)\Psi = 0. \quad (3.2)$$

In correspondence with classical picture, at the quantum level we also have two different cases. In the first case, when $C = -\alpha^2 < 0$, $\alpha > 0$, the quantization of the variables j_μ leads to unitary irreducible representations (UIRs) of the discrete type series D_α^\pm of the group $\overline{\text{SL}}(2, \mathbb{R})$ being the universal covering group of $\text{SL}(2, \mathbb{R})$. In these representations the Casimir operator takes value $J^2 = -\alpha(\alpha - 1)$ substituting the corresponding classical value of the constant C , and the operator J_0 takes the eigenvalues $j_0 = \pm(\alpha + n)$, $n = 0, 1, \dots$, i.e. here we have infinite-dimensional half-bounded representations. In the case when $C = \beta^2 \geq 0$, the quantization leads to the UIRs of the principal continuous series C_σ^ϑ , $\vartheta \in [0, 1)$, $J^2 = \sigma = \beta^2 + 1/4$, and $j_0 = \vartheta + n$, $n = 0, \pm 1, \pm 2, \dots$ (or of supplementary continuous series with $0 < \sigma < 1/4$ [16]), i.e. in this case we have unbounded infinite-dimensional representations in correspondence with classical picture.

Passing over to the rest frame system, $\mathbf{p} = 0$, one can check that equations (3.2) have nontrivial solutions under the coordinated choice of the representation of $\overline{\text{SL}}(2, \mathbb{R})$ and of the value of the spin parameter s . In particular, in the case when Ψ carries the representation of the discrete type series D_α^\pm , and $s = \epsilon\alpha$, $\epsilon = +1$ or -1 , (that corresponds to the special classical case mentioned in the end of the preceding section), eqs. (3.2) have nontrivial solution describing the state with mass m and spin $s = \epsilon\alpha$ [15].

Considering equations (3.2) as field equations, one could try to construct the corresponding field action and then realize the secondary quantization of the theory in order to reveal a spin-statistics relation for fractional spin fields. But, unlike the case of the Dirac equation and equation for the topologically massive vector U(1) gauge field [17], equations (3.2) are completely independent, and, so, they are not very suitable for realizing such a program. Therefore, we arrive at the problem of constructing the set of linear differential equations (by analogy with above mentioned equations), from which equations (3.2) would appear as a consequence of integrability condition (see also refs. [18]-[20]).

Before going over to the consideration of this problem, let us note that in the above mentioned special case, when $s^2 = \alpha^2$ and $J^\mu \in D_\alpha^\pm$, the second equation from the set (3.2) is the (2+1)-dimensional analog of the Majorana equation [21]. This equation describes the quantum states of the model of relativistic particle with torsion [15, 22], and like the Majorana equation itself [21], has solutions in the massive ($p^2 < 0$), massless ($p^2 = 0$) and tachyonic sectors ($p^2 > 0$). Moreover, in the massive sector it has the following mass spectrum: $M_n = m\alpha/|s_n|$, $n = 0, 1, \dots$, where $s_n = \epsilon(\alpha + n)$ is the spin of states. So, from the point of view of the Majorana equation, the role of the Klein-Gordon equation consists in removing the tachyonic and massless states and in singling out from the infinite tower of states the state with highest mass and lowest spin modulus.

The following ‘universal’ vector set of linear differential equations for fractional spin field,

$$V_\mu \Psi = 0, \quad V_\mu = \alpha P_\mu - i\epsilon_{\mu\nu\lambda} P^\nu J^\lambda + \epsilon m J_\mu, \quad \epsilon = \pm 1, \quad (3.3)$$

was proposed in ref. [9]. This set of equations has nontrivial solutions in the case of the choice of the two types of irreducible representations of the group $\overline{\text{SL}(2, \mathbb{R})}$: either unitary infinite-dimensional representations of the discrete type series D_α^\pm ($\alpha > 0$), or $(2j + 1)$ -dimensional nonunitary representations of the discrete type series \tilde{D}_j in the case when $\alpha = -j$, $0 < 2j \in \mathbb{Z}$, and $J^2 = -j(j + 1)$. In the cases when $\alpha = -j = -1/2$ and $\alpha = -j = -1$, and when corresponding 2-dimensional spinor and 3-dimensional vector representations are chosen, the vector system of equations (3.3) is reduced to one equation being the Dirac or Jackiw-Templeton-Schonfeld equation [17], respectively. In all other cases any two of three equations (3.3) can be chosen as a basic set of linear differential equations (3.3) and all the set of three equations is necessary only to have a manifestly covariant set of equations. Equations (3.2) appear as a consequence of the basic equations (3.3) and, as a result, vector system of equations (3.3) describes massive fields carrying irreducible representations of the (2+1)-dimensional Poincaré group $\overline{\text{ISO}(2, 1)}$ characterized by mass $M = m$ and spin $s = \epsilon\alpha$. Moreover, the vector set of equations of the form (3.3) has the following interesting property: it singles out itself only half-bounded infinite-dimensional representations D_α^\pm as suitable for the description of fractional spin fields, and reject the use, for the purpose, of unbounded representations of the continuous series [9]. So, the vector set of equations (3.3) gives some link in the description of fractional spin fields and usual integer and half-integer spin fields, but it is not a minimal set of linear differential equations for describing fractional spin fields. The minimal set of equations, as we can conclude, must contain the set of two equations, and if we want to have a manifestly covariant formulation of the theory, we have to look for a spinor set of linear differential equations.

Such a system of equations can be constructed with the help of the deformed (extended) Heisenberg-Weyl algebra involving the Klein operator [23]. This algebra is given by the

following commutation relation [24]:

$$[a^-, a^+] = 1 + \nu K, \quad (3.4)$$

where K is the Klein operator, defined, in turn, by the relations $K^2 = 1$, $Ka^\pm + a^\pm K = 0$, whereas $\nu \in \mathbf{R}$ is a deformation parameter. The algebra (3.4) has unitary representations in the case when $\nu > -1$ [23]. Let us define now the position Q and momentum Π operators, $a^\pm = (Q \mp i\Pi)/\sqrt{2}$, and note that in the coordinate representation $\Psi = \Psi(q)$, $Q\Psi(q) = q\Psi(q)$, the operator Π can be realized as $\Pi = -i(d/dq + K \cdot \nu/2q)$, whereas K can be considered as a parity operator, $K\Psi(q) = \Psi(-q)$.

Now, let us consider the set of operators

$$L_1 = Q, \quad L_2 = \Pi, \quad J_0 = \frac{1}{4}(a^+a^- + a^-a^+), \quad J_\pm = J_1 \mp iJ_2 = \frac{1}{2}(a^\pm)^2.$$

They satisfy the superalgebra

$$[L_\alpha, L_\beta]_+ = 4i(J\gamma)_{\alpha\beta}, \quad [J_\mu, J_\nu] = -i\epsilon_{\mu\nu\lambda}J^\lambda, \quad [J_\mu, L_\alpha] = \frac{1}{2}(\gamma_\mu)_\alpha^\beta L_\beta, \quad (3.5)$$

where $(\gamma^0)_\alpha^\beta = -(\sigma^2)_\alpha^\beta$, $(\gamma^1)_\alpha^\beta = i(\sigma^1)_\alpha^\beta$, $(\gamma^2)_\alpha^\beta = i(\sigma^3)_\alpha^\beta$. Relations (3.5) mean that the operators L_α and J_μ are the generators of $osp(1|2)$ superalgebra with Casimir operator $C_{osp} = J_\mu J^\mu - \frac{i}{8}L^\alpha L_\alpha$ taking here the value $C_{osp} = (1 - \nu^2)/16$. Moreover, the first and third relations mean that L_α are ‘square root’ operators from the $SL(2, \mathbf{R})$ generators J_μ and they are components of $(2+1)$ -dimensional spinor. Note, that on the subspaces of even, $\Psi_+(q) = \Psi_+(-q)$, and odd, $\Psi_-(q) = -\Psi_-(-q)$, functions the generators J_μ act in an irreducible way realizing representations $D_{\alpha_\pm}^+$ with $\alpha_+ = \frac{1}{4}(1 + \nu) > 0$, $\alpha_- = \alpha_+ + 1/2$, respectively: $J^2\Psi_\pm = -\alpha_\pm(\alpha_\pm - 1)\Psi_\pm$. Note also here that the representations of the series $D_{\alpha_\pm}^-$ can be obtained in a simple way by the substitution $J_0 \rightarrow -J_0$, $J_\pm \rightarrow -J_\mp$. Now, let us consider the spinor set of equations:

$$S_\alpha\Psi = 0, \quad S_\alpha = L^\beta((P\gamma)_{\beta\alpha} + \epsilon m \epsilon_{\beta\alpha}), \quad (3.6)$$

where we suppose that $\Psi = \Psi(x, q)$, $P_\mu = -i\partial/\partial x^\mu$ and $\epsilon_{\alpha\beta} = -\epsilon_{\beta\alpha}$ is a spinor metric tensor. The following relation is valid on the subspace of even functions $\Psi_+(x, q) = \Psi_+(x, -q)$ [25]:

$$(\gamma_\mu)^{\alpha\beta} L_\alpha S_\beta \Psi_+ = V_\mu \Psi_+,$$

where V_μ is given by eq. (3.3). This means that on the subspace of even functions Ψ_+ the spinor system of equations (3.6) has nontrivial solutions describing the states of mass $M = m$ and spin $s = \epsilon\alpha_+ \neq 0$. On the other hand, one can check that on the subspace of odd functions this spinor set of equations has no nontrivial solutions [23].

In conclusion let us point out on the hidden nonlocality of the present constructions. Indeed, the use of half-bounded infinite-dimensional representations for the construction of linear differential equations (describing effectively one-component field in a covariant way [23]) could be associated with the half-infinite nonobservable ‘string’ of the nonlocal anyonic field operators in the approach which uses the statistical Chern-Simons $U(1)$ gauge field [6]. In the case of minimal spinor set of linear differential equations the hidden nonlocality equivalently reveals itself in the dependence of even functions $\Psi_+(x, q) = \Psi_+(x, -q)$ on continuous additional variable $q \in \mathbf{R}$, that effectively correspondence to giving the fractional spin field on some half-infinite ‘string’.

4 Concluding remarks

Starting from the minimal canonical formulation described in section 2, one could try to construct the field action leading to the Klein-Gordon equation being the quantum analog of the only constraint (2.1) of the theory. But the problem consists here in the absence of the representation with the covariant operators X_μ^c to be diagonal. One could use a representation with diagonal operators X_μ^l , but these coordinates have complicated transformation properties with respect to the Lorentz transformations [16], and, therefore, the theory will have manifestly noncovariant character. Moreover, it is necessary to take into account a nontrivial behaviour of the corresponding field $\Psi(x_\mu^l)$ with respect to the Lorentz transformations [18].

So, because of these problems it seems more appropriate to work within a framework of the extended formulation, where the possibility to describe arbitrary spin fields is achieved through the use of the infinite-dimensional representations of the universal covering group of $SL(2, \mathbb{R})$ group. However, here we have still an open problem of constructing the field action which would lead to the system of minimal spinor set of linear differential equations. In this case we have two equations for one (infinite-component [23], or depending on additional continuous argument q) basic fractional spin field. Therefore, it is necessary to introduce into the theory some auxiliary field(s) and the problem is how to realize the extension of the system in some minimal way. Having such field action, we could realize the secondary quantization of the theory to reveal a spin-statistics relation. The hidden nonlocal nature of the present constructions, speculated in the end of the preceding section, can be considered as an indication [6, 7] that a spin-statistics relation indeed can be revealed for fractional spin fields in the group-theoretical approach.

The work was supported by MEC-DGICYT.

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